Let V and W be vector spaces and $T \to W$ be a linear transformation

a) Prove T is one to one if and if T carries linearly indepentent. sets of onto linearly independinte subsets of W

Let $S = \{v_1, v_2, \dots v_k$ linealmple independant of V then $T(S) =$

$\{T(v_1), T(v_2), \dots T(v_k)$ is subset of W

then equation $c_1 T(v_1 + c_2 T(v_2) + \dots + c_k v_k = 0$

first asume T is one to one, then $T(x) = 0$ implies $x = 0$

Hence $T(c_1 v_1 + c_2 v_2 + \dots c_k v_k) = c_1 T(v_1) + c_2 T(v_2) + \dots + c_k T($

implies $c_1 v_1 + c_2 v_2 + , \dots , c_k v_k = 0$

Then $c_1 = c_2 = \dots c_k = 0$.

b) Suppose S is linearly independient since T is one to one, from part (a) we can see that T(S) is linearly independent.

c) first, from β is a basis for V, we have β is linearly independent. And T is one to one we can conclude that $T(\beta)$ generates W. For arbitrary vector w in W, we can v in V such that $T(v) =$ because T is onto. since β basis for V, there exist $c_1, c_2, \dots, c_n$ such that $v = c_1 v$ $\dots + c_n v_n$. then $w = T(v) = T(c_1 v_1 + c_2 v_2 + \dots c_n v_n) =$ $c_1 T(v_1) + c_2 T(v_2 + \dots, c_k T(v_k)$. since w is arbitrary vector w, we conclude that W is generated by $T(\beta)$

Therefore $T(\beta)$ is a basis for W.