

Maurocio Carreón Cristal

Strang, G., Linear Algebra and its applications. 4th Ed.
Cengage Learning 2006.

Capítulo 2

Sección 2.6

Ejercicio 25

25: ¿Cual de estas transformaciones satisfacen
 $T(v+w) = T(v) + T(w)$ y cuales satisfacen
 $T(cv) = cT(v)$?

Usando b) $T(v) = v_1 + v_2 + v_3$

$$\begin{aligned} & T((v_1, v_2, v_3) + (w_1, w_2, w_3)) \\ &= T(v_1 + w_1, v_2 + w_2, v_3 + w_3) \\ &= (v_1 + w_1 + v_2 + w_2 + v_3 + w_3) \\ &= (v_1 + v_2 + v_3) + (w_1 + w_2 + w_3) \\ &= T(v) + T(w) \end{aligned}$$

∴ b) satisfacen $T(v+w) = T(v) + T(w)$

Usando b) $T(u) = v_1 + v_2 + v_3$

$$\begin{aligned}T(K(v_1, v_2, v_3)) &= T(Kv_1, Kv_2, Kv_3) \\ &= Kv_1 + Kv_2 + Kv_3 \\ &= K(v_1 + v_2 + v_3) \\ &= K T(u)\end{aligned}$$

\therefore b) satisfice $T(Kv) = K T(v)$

Usando c) $T(v) = (v_1, 2v_2, 3v_3)$

$$\begin{aligned} & T(v_1, v_2, v_3) + (w_1 + w_2 + w_3) \\ &= T(v_1 + w_1, v_2 + w_2, v_3 + w_3) \\ &= (v_1 + w_1, 2(v_2 + w_2), 3(v_3 + w_3)) \\ &= (v_1 + w_1, 2v_2 + 2w_2, 3v_3 + 3w_3) \\ &= (v_1, 2v_2, 3v_3) + (w_1, 2w_2, 3w_3) \\ &= T(v) + T(w) \end{aligned}$$

\therefore c) satisfice $T(v+w) = T(v) + T(w)$

Usando c) $T(v) = (v_1, 2v_2, 3v_3)$

$$\begin{aligned} & T(kv_1, kv_2, kv_3) \\ &= T(kv_1, kv_2, kv_3) \\ &= (kv_1, 2kv_2, 3kv_3) \\ &= k(v_1, 2v_2, 3v_3) \\ &= kT(v) \end{aligned}$$

\therefore c) satisfice $T(kv) = kT(v)$

El inciso b y c que son b) $T(v) = v_1 + v_2 + v_3$
y c) $T(v) = v_1, 2v_2, 3v_3$ satisfacen
 $T(v+w) = T(v) + T(w)$ y $T(cv) = cT(v)$